HYDRODYNAMICS OF FLOW PAST SPHERES IN A BED

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Substantiation is given for a theoretical-experimental scheme for a coherent, concise generalization of data on the hydraulic resistance of random and regular beds of spherical particles.

There are two standard approaches to describing the hydrodynamics of a flow of liquid or gas through a bed of spherical particles. One approach, involved with the study of flow past a single sphere (external problem), was first suggested by Burke and Plummer [1] and was later developed by the authors of [2-4]. The other approach involves examination of the flow in the channels formed between spheres (internal problem). Representation of a spherical bed as a system of pore channels is closely associated with the works of Blake [5], Kozeny [6], Carman [7], and Ergun [8].

The use of a mathematical model based on the concepts of an external and an internal problem makes it possible to calculate the coefficients of hydraulic resistance for spherical beds. The coefficients calculated on the basis of existing models are close to the experimental values established for random (disordered) beds but differ significantly from those found for regular (ordered) beds of spheres [9].

In this article, we present a theoretical-experimental scheme for a coherent, concise generalization of data on the hydraulic resistance of random and regular beds. We examine the entire range of Reynolds numbers and, accordingly, all possible regimes of flow: viscous, viscous-inertial, and inertial.

It is known that in the course of the transition from purely viscous to purely inertial flow in spherical beds, there is a smooth decrease in hydraulic resistance. The resistance crisis characteristic of flow past a single sphere and connected with the transition from laminar to turbulent flow is not seen. This makes it possible to use the same relation to determine the coefficient of hydraulic resistance of the bed ζ for all flow regimes. This relation takes the form either of the Dupuis—Forheimer formula [9]

$$\zeta = \frac{a}{\mathrm{Re}} + b, \tag{1}$$

or the formula constructed by A. D. Al'shul' [10]

$$\zeta = \frac{a}{\mathrm{Re}} + \sqrt{\zeta b}. \tag{2}$$

We will henceforth use Eq. (1) and determine the coefficients a and b from an analysis of limiting cases involving purely viscous ($\zeta = a/\mathrm{Re}$) and purely inertial ($b = \zeta$) flow regimes.

We assume that the spheres in the bed are under identical hydrodynamic conditions. This widely used assumption allows us to represent the pressure gradient ΔP at the boundaries of the bed in the form [11]:

$$\Delta P = \frac{2}{3}\,\frac{1-\varepsilon}{\varepsilon}\,\zeta\,\frac{\rho\tilde{w}^2}{2}\,\frac{H}{d}. \tag{3}$$

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Thus, in order to be able to use Eq. (3) to determine the hydraulic resistance ΔP of a spherical bed, it is important to substantiate the choice of the characteristic w and determine flow velocity w and determine the resistance coefficient of a sphere under constrained conditions.

The analysis performed in [11] shows that an eddy-free region of flow exists in the bed, regardless of the flow regime. The effect of constraint of the flow on the resistance force associated with the body in eddy-free flow is accounted for by means of the formula

$$F\psi = F_0\psi_0, \tag{4}$$

so that it is best to take the maximum velocity in the bed as the main characteristic. We then introduce the following assumptions as the basis for the proposed mathematical model of flow past a sphere under constrained conditions.

1. We are examining a spherical bed consisting of several independent cells, each of which contains a sphere surrounded by a moving liquid sheath. The cells can be considered independent if there is no exchange of energy between a given cell and the surrounding medium [13]. Friction is absent on the outside surface.

2. The mutual effect of the spheres in the bed is such that vorticity is localized in a thin layer of liquid on the surface of the spheres. Outside of this layer, the flow can be considered irrotational.

3. In the spherical bed, the flow decomposes into jets. The interaction of these jets within the confines of a given cell can be ignored.

We will examine viscous (slip) unbounded flow past a sphere with vorticity localized in a thin layer on the sphere's surface. Potential flow exists outside this layer. The components of velocity on the external boundary of the vortex region appear as follows in spherical coordinates

$$w_\theta^+ = \frac{3}{2} w \sin\theta, \quad w_r^+ = 0. \tag{5}$$

The principle of the minimum of the dissipation of mechanical energy [13] is valid for any viscous flow. In accordance with this principle, energy cannot be localized, such as in a boundary layer. We therefore asume that the thickness of the vorticity layer is so small that energy dissipation in it can be ignored. Such an assumption leads to the model of a vorticity layer with vanishing strain [11]. This model will be reduced here to representation of a boundary flow in the form of superimposed eddies moving over the surface of the sphere. Having chosen the center of the sphere as the pole of motion in this case, we obtain the following for the velocity on the external boundary of the vorticity layer in accordance with the Cauchy—Helmholtz formulas

$$w_\theta^+ = \frac{1}{2} r^+\omega, \quad w_r^+ = 0, \tag{6}$$

where ω is the vorticity of the layer.

Combining the internal (6) and external (5) solutions leads to the condition

$$\omega = 3w \sin\theta / r^+. \tag{7}$$

We will make use of the following technique from the theory of small perturbations: we will transfer the boundary conditions from the real surface to the undisturbed surface. Such a transfer introduces an error of second-order smallness in the boundary conditions. In accordance with this, we will ignore the thickness of the vortex region on the surface of the spheres.

The work done by the force caused by the stress on the surface of the sphere per unit of time Fw must be equal to the rate of energy dissipation. The latter quantity can be calculated on the basis of the well-known theorem from the dynamics of viscous fluids [14]

$$\dot{E} = 2\mu \int_S \left(w_\theta \omega - \frac{1}{2} \frac{\partial w_\theta^2}{\partial r} \right) dS. \tag{8}$$

On the potential-flow side, $\omega = 0$ [15]:

$$\dot{E} = -\mu \int_S \frac{\partial w_\theta^2}{\partial r} dS = -6\pi d\mu w^2. \tag{9}$$

In deriving Eq. (9), Batchelor [15] failed to note that energy dissipation is assumed to be absent in the vorticity localization region. Without this assumption, it is impossible to ignore the first term in the right side of Eq. (13). This was also pointed out by G. Yu. Stepanov in [15]. A detailed analysis of the significance of this assumption can be found in [11].

In accordance with (5), the degree of compression of the potential flow is equal to $\psi_0 = 3/2$. With allowance for (4) and (9), we find that under constrained conditions each sphere is acted upon by the force

$$F = F_0 \psi_0^2/\psi^2 = \frac{8}{3} \pi d\mu w/\psi^2, \tag{10}$$

while the resistance coefficient, calculated from the maximum flow velocity in the neighborhood of the sphere, is equal to

$$\zeta = \frac{64}{3\mathrm{Re}} = \frac{21{,}3}{\mathrm{Re}}. \tag{11}$$

This formula makes it possible to calculate the coefficient of hydraulic resistance of random and regular spherical beds in the case of a viscous flow regime. It serves as the basis for calculation of the resistance coefficient from the maximum fluid velocity in the bed and calculation of the Reynolds number from the velocity in front of the bed. A comparison was made in [11] of the resistance coefficients calculated from Eq. (11) and determined from well-known experimental data in accordance with the formula

$$\zeta = \frac{2}{3} \frac{\varepsilon \psi^2}{1-\varepsilon} \frac{2\Delta P}{\rho w^2} \frac{d}{H}. \tag{12}$$

The theoretical values of the resistance coefficient agree satisfactorily with the experimental values.

To determine the resistance coefficient for a purely inertial flow regime, we will assume that most of the energy loss of the flow within a spherical cell is due to the expenditure of energy on expansion of the jet. The losses from compression, rotation, and friction will be ignored.

For attached flow past the sphere, the expansion of the flow within the cell is due only to its geometry. In this case, the resistance coefficient can be determined in accordance with the approach taken by M. A. Gol'dshtik [2]. The coefficient is thus determined from the viewpoint of the external hydrodynamic problem [11]:

TABLE 1. Comparison of Theoretical and Experimental Data

Type of arrangement [16]	ε	ψ	Calculation a	Calculation b	Expt.[19] a	Expt.[19] b	ε	ψ	b calculation	b expt. [20]
Cubic	0,4764	0,214		0,23	22	0,26	0,483	0,224	0,23	0,23
Orthorhombic		0,093		0,20	24	0,28	0,415	0,123	0,33	0,33
Rhombic*	0,3954			0,20	22	0,25	0,403	0,224	0,19	0,30
Rhombic*		0,214		0,36	22	0,38			0,36	
Birhombic			21,3	0,20	20	0,24	0,324	0,123	0,27	0,25
Birhombic	0,3019			0,17	20	0,20			0,23	
Tetrahedral		0,093		0,17	22	0,16	0,293	0,135	0,22	0,23
Tetrahedral*	0,2595			0,17	22	0,16			0,22	
Octahedral				0,15	22	0,20	0,268	0,135	0,16	0,16
Disordered	0,4	0,17		0,32	21	0,30				

*Beds with blocked channels.

$$b = 2\frac{\psi}{1-\psi}\left(1-\frac{\psi}{\psi'}\right). \tag{13}$$

An analysis of the geometric characteristics of spherical beds was made in [16]. Among the main relations are the following:

$$\psi' = 1 - 6(1-\varepsilon)(h/d)^2(1-h/d), \tag{14}$$

$$h/d = \frac{2}{3}\frac{1-\psi}{1-\varepsilon}. \tag{15}$$

With allowance for the Leibenzon-Bogoyavlenskii formula $\psi = 0.61\varepsilon^{1.4}$ [17], Eq. (13) is simplified to the following form [18] in the range $\varepsilon = 0.26$-0.48 for random spherical beds:

$$b = 1{,}25\varepsilon^{1{,}5}. \tag{16}$$

The model of attached flow cannot be used for all sphere arrangements. It is valid only for beds with blocked channels. In beds of spheres with through channels, local compression and subsequent expansion of the jets cause their separation from the surface of the spheres [17]. Under these conditions, the resistance coefficient of a sphere in the bed is determined not only by the geometric characteristics of the bed, but also by the angle of expansion of the jet. These factors can be taken into account in a model of jet flow in a spherical bed from the viewpoint of the internal hydrodynamic problem if, following R. G. Bogoyavlenskii [17], we take $\beta \approx 15°$ for the angle of expansion of the jets in the cells.

In accordance with the Bord–Carnot formula, the resistance coefficient for a jet in a cell calculated on the basis of the maximum velocity in the jet is equal to

$$\lambda = (1-f)^2. \tag{17}$$

For regular beds with through channels, the degree of expansion of the jet f can be calculated from geometric relations [11]

$$l = r + r_1 - r_0 - 2r(h/d)\,\mathrm{tg}\beta,\; m = \frac{\sqrt{r^2 + \mathrm{tg}^2\beta - l^2} - l\,\mathrm{tg}\beta}{1+\mathrm{tg}^2\beta}, \tag{18}$$
$$r_2 = r_0 - m\,\mathrm{tg}\beta + 2r(h/d)\,\mathrm{tg}\beta,\; f = (r_0/r_2)^2.$$

In a hexagonal arrangement of spheres, $r_0 = 0.227r$; $r_1 = 0.153r$. In a cubic arrangement, $r_0 = 0.523r$; $r_1 = 0.414r$.

The resistance coefficients of a jet λ and a sphere ζ in a bed are connected by the relation

$$\zeta = \frac{2}{3}\frac{\varepsilon}{1-\varepsilon}\frac{d}{h}\lambda. \tag{19}$$

It is evident from the table that the theoretical values of the coefficients a and b agree satisfactorily with the experimental values. The values of b for different beds change within the range 0.16-0.38 and are grouped around the resistance coefficient for a single sphere. The latter is equal to $b_0 = 0.45\ \psi_0^2 = 0.2$ in apurely intertial flow regime [11].

Thus, model (11-13), (16-19) has made it possible to standardize the calculation and representation of data on the hydraulic resistance of random and regular spherical beds within a broad range of Reynolds numbers.

NOTATION

ζ, λ, coefficients of hydraulic resistance of a sphere and a jet in a bed; d, r, diameter and radius of sphere; H, h, height of bed and distance between adjacent rows of spheres; ℓ, m, r_0, r_1, r_2, geometric parameters of a cell with through channels; ε, porosity of the bed; f, degree of expansion of the jet in the cell; β, angle of expansion of the jet; ψ', ψ, relative maximum and minimum through sections in the bed; ψ_0, degree of constraint of the flow in the neighborhood of the sphere with unbounded flow; w, w_{max}, velocity of the liquid in front of the bed and maximum velocity in the bed; w_θ, w_r, components of fluid velocity in spherical coordinates with the pole at the center of the sphere; ω, vorticity; F, F_0, forces due to stresses on the surface of the sphere in the bed and in the unbounded flow; $\dot{E}$, rate of energy dissipation; $Re = \rho w d/\mu$, Reynolds number; ρ, μ, density and absolute viscosity.

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WAVE REGIME OF CONSOLIDATION OF A POROUS COMPRESSIBLE MEDIUM

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An analytical solution in the form of a compression wave is found to the problem of the consolidation of a porous medium. Questions relating to the validity of the solution are examined.

In the study of the problem of the consolidation of a viscous compressible medium in the theory of hot pressing [1, 2], it is customary to ignore the inertial and nonsteady terms in the equations of motion and to replace these equations by simpler conditions of equilibrium [3-5]. This simplication is usually connected with small Reynolds numbers Re. The smallness of Re for the hot pressing of hard alloys is based on approximate calculations [4]. In this case, the initial variation of density in the material is important only in regard to the scale factor and has no effect on the character of the dependence of density on time. The perturbation from the piston is transmitted instantaneously to all discrete volumes of material. Such a consolidation regime has been called the regular regime [5].

Strictly speaking, the validity of ignoring inertial and nonsteady terms in the equations of motion depends not only on the smallness of Re, but also on the value of the partial derivatives of velocity with respect to the coordinates and time. At the same time, the inertia of the medium itself accounts for several fundamental characteristics of the process. It is important that the perturbation from the piston is not transmitted instantaneously to all discrete volumes in such media. Because this is the case, the preconditions are established for the formation of a compression wave in the porous medium. In connection with this, it is interesting to examine the problem of the compression of a porous medium with allowance for its inertia. In the present investigation, we seek to study the possibility of the occurrence of consolidation regimes other than the regular regime by solving the problem in the form of a compression wave. Here, we make use of the concept of intermediate asymptotes [6]. The solution of the problem of the compression of a porous medium with allowance for inertial and nonsteady terms allowed us to find the necessary conditions for occurrence of the regular consolidation regime — the conditions under which we can ignore the inertia of the medium. It is shown that the realization of both transitional and wave regimes of consolidation is possible. Distributions of density, velocity, and stress are found for materials which undergo consolidation in the wave regime.

Formulation of the Problem. We will examine the axial compression of a viscous porous medium under the influence of a piston moving from right to left. The motion of the medium during its consolidation is described by the equations of continuity and motion together with rheological relations and boundary conditions

$$\partial\rho/\partial t + \partial(\rho U)/\partial x = 0, \tag{1}$$

$$\rho\rho_1(\partial U/\partial t + U\partial U/\partial x) = \partial\sigma/\partial x, \tag{2}$$

$$\sigma = (4\eta/3 + \mu)\,\partial U/\partial x = \sigma_0 \frac{\rho^m}{1-\rho}\,\partial U/\partial x, \tag{3}$$

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